# UNITED STATES AIR FORCE
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# ON THE NON-DISPERSIVENESS OF A COMPOSITE OF NON-LAYERED DISPERSIVE MEDIA

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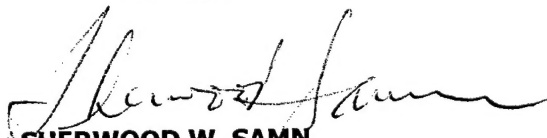
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We studied the question whether a composite of non-layered dispersive media be effectively non-dispersive using a line source. Since the recovery of the reflection coefficient of a medium (half space) from observed reflected waves in the line source case is not as direct as in the plane wave case, an alternate approach is suggested to help decide if a given medium is non-dispersive. This approach which is based on a simple measure is used in several problems, both theoretical and FDTD simulations.

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# Contents

# List of Figures

# On the Non-Dispersiveness of a Composite of Non-Layered Dispersive Media

## I.   Introduction

A conjecture has recently been made [8] in the context of detection and identification of visually obscured objects that a composite of many non-layered dispersive material is effectively non-dispersive: somewhat akin to the central limit theorem in statistics. This is the main motivation for the following study.

Intrinsically a medium is either dispersive or non-dispersive. Since most realistic media are conductive (and therefore dispersive) to some extent, we will say, for our purpose here, a medium is non-dispersive, if it is apparently or effectively non-dispersive. In particular, we will say a medium is non-dispersive if waves reflected off the medium behave like those reflected off a (theoretically) non-dispersive medium. Another characterization of the non-dispersiveness of a medium can be defined in terms of transmitted waves, although in practice (for example, in the setting of ground penetrating radars) they are not readily available.

This leads us to the question of how to characterize waves reflected off non-dispersive media. For plane waves incident the reflection coefficient, the ratio of the energy or amplitude in the incident wave to that in the reflected wave, can be calculated. Moreover, it is a function only of the angle of incident and the electromagnetic properties of the media and is therefore frequency independent if the electromagnetic parameters are independent of frequency. Thus for plane wave incident we have a way to characterize waves reflecting off non-dispersive media. However, a plane wave (a 1-D concept) is only an idealization that does not really exist. A line source (a 2-D concept) instead is more realistic. In this report we will discuss a way to determine if a half-space medium is non-dispersive using information in the reflected waves, when the incident wave is a line source.

Once we have identified the type of medium we have at hand, for example, that it is non-dispersive, we can proceed to identify its electrical parameters more easily. This approach can be generalized to conductive media that are otherwise non-dispersive and may lead to a method that would complement dielectric spectroscopy [4], an important component in electromagnetic dosimetry.

In Section II of this report, we will describe briefly the Finite-Difference Time-Domain (FDTD) computer program we used in this study. A measure of non-dispersiveness is then proposed in Section III. Section IV summarises simulations we have performed to validate this measure and also includes an application to a non-layered dispersive medium.

## II. FDTD

### II-1. Overview

In recent years, FDTD has become an extremely popular and useful tool to model transient electromagnetic propagation. We also used it here in our study. The backbone of our computer program is the standard 2D Yee FDTD code [10].

One of the drawbacks of FDTD was the need to use a large computation domain to eliminate artificial reflections from the boundary of the computation domain. This translates to large computer memory requirement. With the discovery of Perfectly Matched Layer (PML) technology [1] in 1994, this is no longer a problem. The basic idea of PML is to surround the (not necessarily large) computation domain with a relatively thin layer of judiciously chosen material (artificial or otherwise) that will absorb the incoming waves and thus not reflect them back into the computation domain. While the 2D problem we are considering here is not computer memory bounded, nevertheless we have augmented our basic program with PML in anticipation of its future extension to 3D. The version of PML we have finally adopted is described in detail in [5] by Gedney. There are other

2

PML approaches one could adopt, for example the stretched coordinates approach advanced by Chew [3]. However, Gedney's approach seems to be easier to implement.

FDTD is a time domain approach. On the other hand, dispersive media are naturally characterized in the frequency domain. Hence it is not as straight forward to model dispersive media in FDTD as it is in, for example, an integral equation approach in the frequency domain. There are two common approaches to handling dispersive media in a FDTD code: one is a convolution approach [6] and the other is a auxiliary differential equation approach [9]. We adopted the latter in our code.

In the rest of this section we will highlight some of the advanced features of the code.

## II-2.  Maxwell's Equations

The well-known Maxwell's equations for the electromagnetic field propagation consist of six scalar equations involving the three components of the **E** field and three components of the **H** field. Assuming the media are non-magnetic, the main equation that contains possible dispersion parameters is

$$\nabla \times \mathbf{H} = \dot{\mathbf{D}} + \mathbf{J} \tag{1}$$

Or, in the frequency domain, assuming linear isotropic media,

$$\nabla \times \mathbf{H} = -j\omega\epsilon_o \hat{e}_r(\omega)\,\mathbf{E} \tag{2}$$

where $j$ is $\sqrt{-1}$, $\hat{e}_r$ is the relative complex permittivity, and $\epsilon_o$ is the free space permittivity.

In a 2D problem, we assume all six components are independent of one of the coordinates. This allows the six original equations to be separated into two groups, each consists of three equations and involves exactly three field components. Furthermore, one can solve each group separately. For our purpose here, we considered only the group that consists of one **E** component and two **H** components, the so-called TM case. For this case, Equation (1) reduces to one scalar equation.

## II-3. PML

The PML used here is made up of a thin layer of an artificial material and surrounds the actual computation domain (henceforth called the interior). The "electromagnetic" field in the PML does not really satisfy the Maxwell's equations. For the ease of implementation, it is desirable that the field in the PML and the field in the interior both satisfy equations of the same form. This is accomplished by extending the form of the Maxwell's equations in the interior. In particular, Equation (2) is modified to

$$\nabla \times \mathbf{H} = -j\omega\epsilon_o \hat{e}_r(\omega)\, \mathbf{S}\, \mathbf{E} \qquad (3)$$

where

$$\mathbf{S} = \left( \frac{s_{i+1} s_{i+2}}{s_i} \delta_{i,j} \right),$$

$$s_i = \text{functions of space coordinates, } i = 1, 2, 3$$

$$s_j = s_{j-3}, \quad j = 4, 5$$

$$\delta_{i,j} = \text{Kronecker delta}$$

In the interior, we clearly must require $\mathbf{S}$ to be the identity matrix. Hence, in the interior, $s_i = 1$ for $i = 1, 2, 3$. This reduces Equation (3) to Equation (2). In the PML, the function $s_1(x, y, z)$ (and similarly for $s_2$ and $s_3$) may be chosen as

$$s_1(x, y, z) = \kappa_x(x, y, z) + j\frac{\sigma_x(x, y, z)}{\omega\epsilon_o}$$

where

$$\kappa_x(x, y, z) = 1 + \kappa_{max}\left(\frac{\zeta(x, y, z)}{d}\right)^m$$

$$\sigma_x(x, y, z) = \sigma_{max}\left(\frac{\zeta(x, y, z)}{d}\right)^m$$

Here $\zeta(x, y, z)$ is the distance between the point $(x, y, z)$ and the interior in the x-direction and $d$ is the thickness of the PML slab. Typical values for the PML parameters $\kappa_{max}, \sigma_{max}, m,$ and $d$ are

4

$$m = 1$$
$$\kappa_{max} = 1$$
$$\sigma_{max} = 3.5$$
$$d = 10\,\triangle x \quad (\triangle x \text{ is the FDTD grid size })$$

In most cases the performance of this PML is adequate, especially if only the total field is sought. However, in some cases in which the *desired* reflected waves from the medium, calculated by subtracting the incident field from the total field, are small compared to the total field, the *undesired* reflected waves from the boundary of the computation domain, which are still relatively small compared to the total field, may no longer be negligible compared to the *desired* reflected field. In those cases, one may try to fine tune the PML parameters to reduce the *undesired* reflected waves further, or, for expediency, enlarge the computation domain. As mentioned before, this is generally not a severe problem for 2D problems, especially if one is only interested in transient phenomena, since *undesired* reflected waves originated from the boundary of the computation domain would not have time to contaminate the *desired* reflected waves in the region of interest.

## II-4.  Dispersion

The computation in Equation (3) can be decomposed into three steps as follow:

$$\nabla \times \mathbf{H} = -j\omega \mathbf{S}_1\,\mathbf{P}$$
$$\mathbf{P} = \hat{e}_r(\omega)\,\mathbf{D} \qquad (4)$$
$$\mathbf{D} = \epsilon_o\,\mathbf{S}_2\,\mathbf{E}$$

where $\mathbf{S} = \mathbf{S}_1\mathbf{S}_2$. In particular,

$$\mathbf{S}_1 = \left(s_{i+1}\,\delta_{i,j}\right)$$

$$\mathbf{S}_2 = \left( \frac{s_{i+2}}{s_i}\, \delta_{i,j} \right)$$

In the conventional FDTD, one would update $\mathbf{E}$ using the latest values of $\mathbf{H}$. Similarly, as suggested by the 3-step decomposition above, one would update $\mathbf{P}, \mathbf{D}$, and $\mathbf{E}$ in turn using the latest values of $\mathbf{H}$. Since the dispersiveness of the medium only affects the middle equation in the decomposition in Equation (4), we will only address the middle equation here. Thus, given the latest values of $\mathbf{P}$, we need a method to update $\mathbf{D}$. We implement this step via the auxiliary differential equation approach. In particular, we use an approach similar to [Korner, 1997] who treated Lorentz media. Here we consider a general Debye model of the form:

$$\hat{e}_r(\omega) = \epsilon_\infty + j\frac{\sigma_o}{\omega\epsilon_o} + \sum_{k=1}^{n} \frac{\alpha_k}{1 - j\tau_k\omega} \tag{5}$$

$$= \sum_{k=0}^{n+1} \chi_k \tag{6}$$

where $n = 3$ and

$$\chi_o = \epsilon_\infty$$
$$\chi_1 = j\frac{\sigma_o}{\omega\epsilon_o}$$
$$\chi_{k+1} = \frac{\alpha_k}{1 - j\tau_k\omega}, \qquad k = 1,\ldots,n$$

We introduce the new variables $\beta_{k,i}$ defined by

$$\beta_{k,i} = \chi_k D_i, \quad k = 0,\ldots,n+1, \ i = 1,2,3$$

where $D_i$ are the components of $\mathbf{D}$. Then after some simple algebra, one obtains for each $i = 1,2,3$ a system of $n + 2$ equations

$$P_i = \frac{\chi_o\beta_{k,i}}{\chi_k} + \sum_{j=1}^{n+1} \beta_{j,i}, \quad k = 1,\ldots,n+1$$

$$D_i = (P_i - \sum_{j=1}^{n+1} \beta_{j,i})/\chi_o$$

6

In the time domain, the first $n + 1$ equations (for each fixed $i$) become a system of $n + 1$ ordinary differential equations in the $n + 1$ unknowns $\beta_{k,i}$, $k = 1, \ldots, n + 1$. The solution of this system is then used to update $D_i$, using the last equation.

## II-5.   The Line Source

To complete the description of the simulation method, we need to discuss the generation of the incident wave. As mentioned above, we assume the incident wave is generated by a line source. In particular, the line source points into the plane and is parallel to the interface between free space and medium, which is taken to be a half space. The line source is driven by a Blackman-Harris pulse (type 1) with a typical central frequency of 600 MHz. Its time profile and amplitude spectrum (theoretical and calculated) are shown in Figure 1 and Figure 2 respectively.
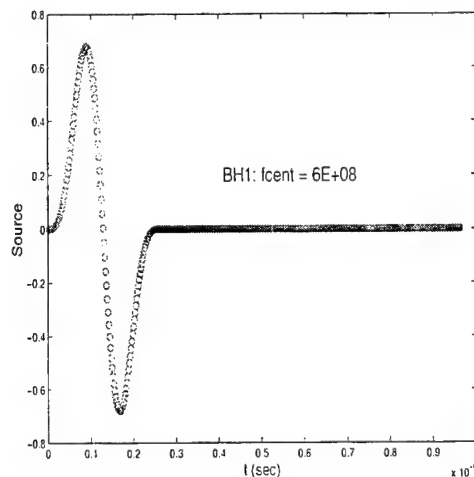


Figure 1: Line Source: Blackman-Harris Type 1.

# III.   A Simple Measure for Non-Dispersiveness

The reflected waves from a half space due to a line source located at the origin, in the frequency domain, for most of the well-known cases have the
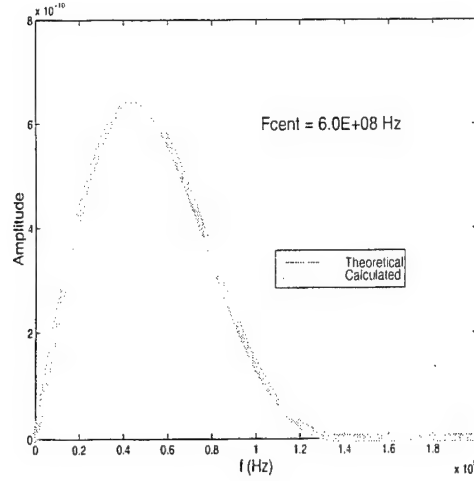
Figure 2: Amplitude of the Fourier Transform of the Blackman-Harris Type 1.

representation [2]:

$$E^R(\omega, x, y) = -\frac{\omega \mu_o I(\omega)}{4\pi} F(\omega, x, y) \tag{7}$$

$$F(\omega, x, y) = \int_{-\infty}^{\infty} \frac{R(\omega, k_x, \ldots)}{k_{1y}} e^{i\phi(\omega)} dk_x \tag{8}$$

$$\phi(\omega) = \omega\{k_x x + k_{1y}(y + 2d_1)\} \tag{9}$$

$$k_{1y} = \sqrt{k_1^2 - k_x^2} \tag{10}$$

Here $(x, y)$ is the observation point, $d_1 > 0$ is the distance between the line source and the half space $(y \leq -d)$, $I(\omega)$ is the spectrum of the line source, and $k_1$ is the wave number of medium 1, which is taken to be free space here.

The "reflection coefficient" $R(\omega, k_x, \ldots)$ is generally a complicated function of the medium properties. In the simplest case of a homogeneous, non-dispersive, non-conducting, non-magnetic half-space, $R$ takes the familiar form

$$R = \frac{k_1 - k_2}{k_1 + k_2} \tag{11}$$

where $k_2$ is the wave number in the half space. Here $R$ is independent of frequency.

8

As in the plane wave case, it is natural to measure the non-dispersiveness of a medium by the degree of frequency independence of $R$, e.g. the mean squared error of $|R|$ over a given frequency range. Unfortunately, $R$ is not easily recoverable from the reflected wave.

If $R$ is hard to calculate, the next choice to measure the non-dispersiveness of a medium is to use the integral of $R$ or $F(\omega, x, y)$. However, even in simplest non-dispersive case, the integral would generally be frequency dependent, as the frequency appears in the exponent inside the integral.

The ratio of reflected wave to incident wave used in the plane wave case does not work either in the line source case as the ratio generally is also dependent on frequency even when the half space is non-dispersive.

The analysis in the plane wave case is relatively simple because there a quantity, the reflection coefficient, that (1) can be calculated and (2) has an invariance property (here with respect to frequency) when the half space is non-dispersive. For the line source case, we would also like to find a quantity that we can calculate and which has some invariance property whenever the half-space is non-dispersive.

We noticed that while the integral in Equation (8) is generally frequency dependent, it has a special property when R is frequency independent, namely

$$F(\omega, x_1, y_1) = F(\omega \frac{x_1}{x_2}, x_2, y_2)$$

or, equivalently,

$$F(\omega, x_2, y_2) = F(\omega \frac{x_2}{x_1}, x_1, y_1) \tag{12}$$

if $(x_1, y_1)$ and $(x_2, y_2)$ are judiciously chosen. In particular, if $(x_1, y_1)$ and $x_2$ are given, then $y_2$ is the unique value such the three points: image of the source, $(x_1, y_1)$ and $(x_2, y_2)$, are collinear. In other words, if a point $(x_1, y_1)$ is given, then *all* the values of F on the line joining $(x_1, y_1)$ and the image of the source are related.

More generally, let $\lambda$ be any real number. A point $(\zeta, \eta)$ is said to be

$\lambda$-related to a point $(x, y)$ if

$$
\begin{aligned}
\zeta = \zeta(x, \lambda) &= \lambda x, \text{ and} \\
\eta = \eta(y, \lambda) &= \lambda y + 2d(\lambda - 1)
\end{aligned}
$$

A point $(\zeta, \eta)$ is said to be related to a point $(x, y)$ if it is $\lambda$-related to $(x, y)$ for some $\lambda$.

We will always assume without saying that the points in question are always in free space, as it is here where the reflected waves are defined. This will place some obvious restrictions on $\lambda$. It readily follows that if $(x_\lambda, y_\lambda)$ is $\lambda$-related to the point $(x, y)$ and if the half space is non-dispersive, then

$$
F(\omega, x, y) = F(\omega/\lambda, x_\lambda, y_\lambda)
$$

Or equivalently,

$$
F(\omega, x_\lambda, y_\lambda) = F(\lambda\omega, x, y) \tag{13}
$$

For example, if $(x_2, y_2)$ is 2-related to $(x_1, y_1)$, then

$$
F(\omega, x_2, y_2) = F(2\omega, x_1, y_1).
$$

An equivalent form of Equation (13) which is amenable to calculation is

$$
\frac{E^R(\omega, x_\lambda, y_\lambda)}{I(\omega)} = \frac{E^R(\lambda\omega, x, y)}{\lambda\, I(\lambda\omega)} \tag{14}
$$

where $(x_\lambda, y_\lambda)$ is $\lambda$-related to $(x, y)$

If we define

$$
G(\omega; x, y) := \frac{x E^R\left(\frac{\omega}{x}, x, y\right)}{I\left(\frac{\omega}{x}\right)}
$$

then a more symmetric form of Equation (14) is

$$
G(\omega; x, y) = G(\omega; x_\lambda, y_\lambda) \tag{15}
$$

Again, Equation (15) holds if the half-space in non-dispersive.

We can now propose a measure M of effective non-dispersiveness based only on the wave reflected from the half space. A natural measure is the

10

mean squared error between the two functions appearing in Equation (15), obtained on two related observation points as defined above, over a given frequency range depending on frequency content of the source:

$$M(E^R, x, y, \lambda) = \frac{1}{|f_2 - f_1|} \int_{f_1}^{f_2} |G(f; x, y) - G(f; x_\lambda, y_\lambda)| \, df$$

In application, one basically needs to calculate the Fourier Transforms of the reflections measured at two related points and divide the resulting spectra by the spectrum of the source. For numerical consideration, the range $[f_1, f_2]$ is chosen where the spectrum of the source is significant, say, over the band-width.

To avoid taking Fourier Transforms, a time domain version can be formulated. In this version, two different but related *sources* are used twice. The reflected waves measured at two related points again are related. Their difference can again provide a measure of the effective non-dispersiveness of the half space. In particular, for a given line source $I(t)$, we define a modified source $I_\gamma(t)$ by

$$I_\gamma(t) := \frac{1}{\gamma} I(\gamma t)$$

If we denote $r(t; x, y, I)$ as the reflection observed at the point $(x, y)$ due to a line source $I(t)$ located at the origin, then, assuming the half-space is non-dispersive, we can readily show:

$$r(x_1 t; x_1, y_1, I_\lambda) = r(x_2 t; x_2, y_2, I) \tag{16}$$

Again, $(x_1, y_1)$ and $(x_2, y_2)$ are any two related points as described above.

## IV.   Numerical Experiments

### IV-1.   Theoretical: Non-Dispersive vs Dispersive Half Space

We investigated numerically the integral defined by $F$ on which our measure is based. In particular, we want to know if the measure or equivalently $F$ can

11

distinguish simple dispersive media from non-dispersive ones. We studied a simple dispersive half space consisting of only soil with 2.2% moisture. Its relative complex permittivity, Equation (5), has the following parameters [7]:

$$\begin{aligned}
\epsilon_\infty &= 3.45 & \sigma_o &= 1.59E - 05 \\
\alpha_1 &= 12.38 & \tau_1 &= 2.37E + 06 \\
\alpha_2 &= 2.40 & \tau_2 &= 1.05E + 08 \\
\alpha_3 &= 0.27 & \tau_3 &= 2.71E + 10
\end{aligned}$$

The dependence of its complex permittivity on frequency is depicted in Figure 3 and in Figure 4. The relationship between $\hat{e}_r$ in Equation (5) and the real-valued quantities $er$ and $\sigma$ in the figures are:

$$\hat{e}_r(\omega) = er(\omega) + j\frac{\sigma(\omega)}{\epsilon_o \omega} \tag{17}$$

It readily follows that

$$\begin{aligned}
er(\omega) &= \epsilon_\infty + \sum_{k=1}^{3} \frac{\alpha_k}{1 + (\tau_k \omega)^2} \\
\sigma(\omega) &= \sigma_o + \omega^2 \epsilon_o \sum_{k=1}^{3} \frac{\alpha_k \tau_k}{1 + (\tau_k \omega)^2}
\end{aligned}$$

Using this complex permittivity in $R$ as defined in Equation (11), we numerically integrated Equation (8) to obtain $F(\omega, x, y)$ at two distinct but related points, as described in Section III. The result is shown in Figure 5 for this dispersive case. The top plot displays $F(\omega, x, y)$ and $F(\omega, x_\lambda, y_\lambda)$, where $\lambda$ has been chosen to be 2. If the half space were non-dispersive, we should have, according to Equation (13), $F(\omega, x_\lambda, y_\lambda) = F(\lambda\omega, x, y)$. The bottom plot in Figure 5 tries to validate this relationship by displaying the graphs of $F(\omega, x, y)$ and $F(\frac{\omega}{\lambda}, x_\lambda, y_\lambda)$. The two should coincide if the half
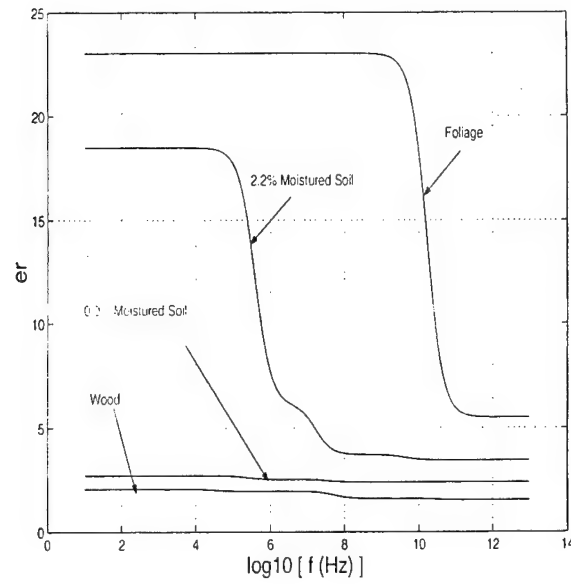
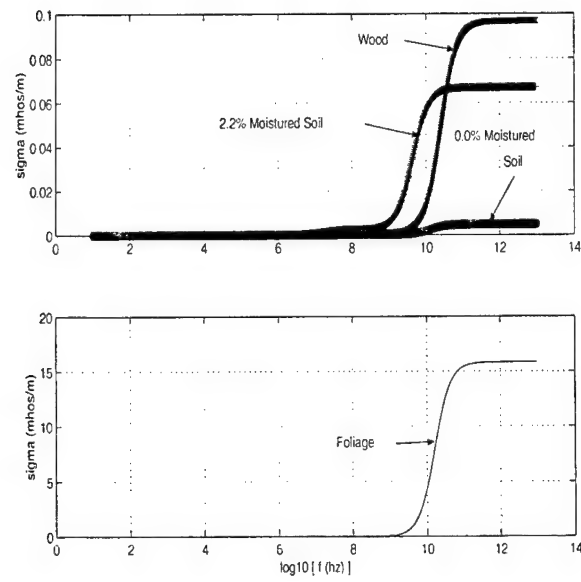12

Figure 3: Er, the real part of the complex permittivity.
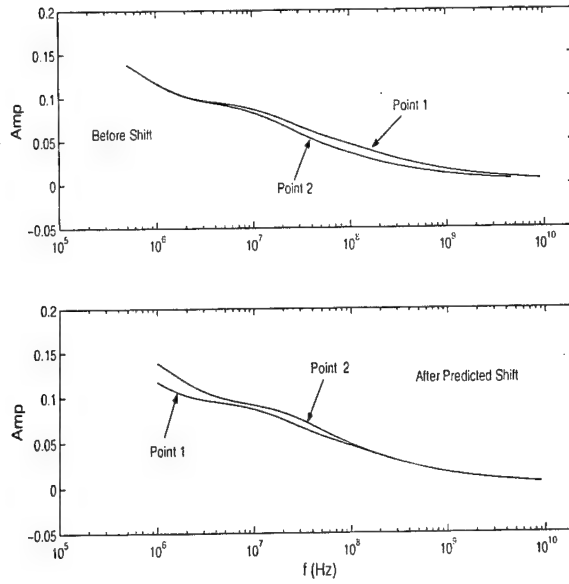


Figure 4: $\sigma$ in the complex permittivity.

Figure 5: The Dispersive Case. Top: $|F(\omega, x, y)|$ and $|F(\omega, x_\lambda, y_\lambda)|$, $\lambda = 2$. Bottom: $|F(\omega, x, y)|$ and $|F(\frac{\omega}{\lambda}, x_\lambda, y_\lambda)|$.

space were non-dispersive. Clearly they do not. Hence, this half space cannot be non-dispersive, as we expect.

Next, we apply the measure to a non-dispersive case. We repeat the same calculation for an "averaged" 2.2% moisture non-conducting soil, i.e.,

$$\epsilon_\infty = 3.45 \quad \text{and} \quad \sigma_o = 0$$

The result is shown in Figure 6 which displays the same quantities that were shown in Figure (5). After scaling $F(\omega, x_\lambda, y_\lambda)$ by the amount suggested by Equation (13), the F-values at the two points coincide (bottom plot in Figure 6), suggesting the half space is non-dispersive.

This result together with the previous one show the capability of the simple $F(\omega, x, y)$-based measure to resolve the question of non-dispersiveness. However, it should be noted that $F(\omega, x, y)$ is not directly measurable in practice.
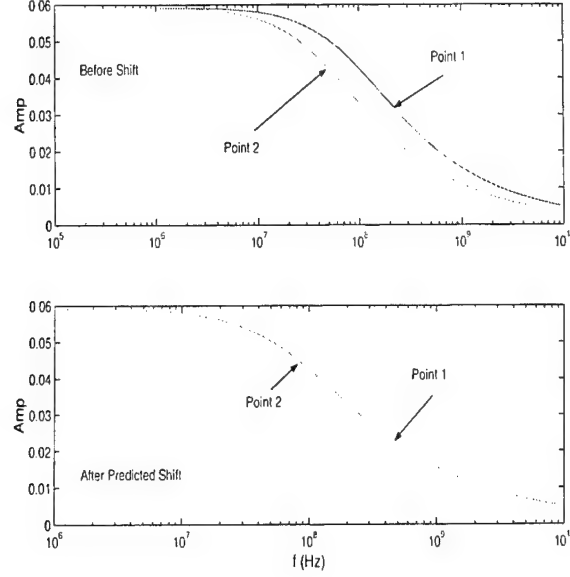
14

Figure 6: Non-Dispersive Case. Top: $|F(\omega, x, y)|$ and $|F(\omega, x_\lambda, y_\lambda)|$, $\lambda = 2$. Bottom: $|F(\omega, x, y)|$ and $|F(\frac{\omega}{\lambda}, x_\lambda, y_\lambda)|$.

## IV-2. FDTD: Non-Dispersive vs Dispersive Half Space

Using the FDTD program outlined in Section II, we conducted two similar numerical experiments based only on quantities that are directly measurable: namely, the reflected waves from the half space.

While $F(\omega, x, y)$ can not be measured directly, it can be estimated (up to a factor of $\omega$) by the quotient of the spectrum of the reflected wave measured at $(x, y)$ and that of the line source (see Equation (7)). We conducted two numerical experiments: one with the non-dispersive half space and one with the dispersive half space used in the previous sub-section. The results are shown in Figure 7 and Figure 8

In these experiments, we have again taken $\lambda = \frac{x_2}{x_1} = 2$. The top plots in Figure 7 and Figure 8 display the graphs of $\frac{E^R(\omega,x,y)}{I(\omega)}$ and $\frac{E^R(\omega,x_\lambda,y_\lambda)}{I(\omega)}$. The bottom plots in each of these figures display the graph of the scaled quotient $\frac{E^R(\lambda\omega,x,y)}{\lambda I(\lambda\omega)}$ and the (unscaled) quotient $\frac{E^R(\omega,x_\lambda,y_\lambda)}{I(\omega)}$. According to Equation 14, the latter two graphs should be the same if the half space is non-dispersive. Our simulations indeed verify this.
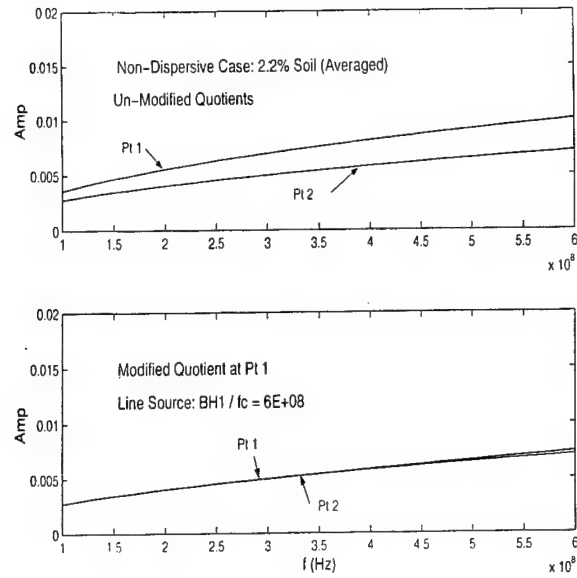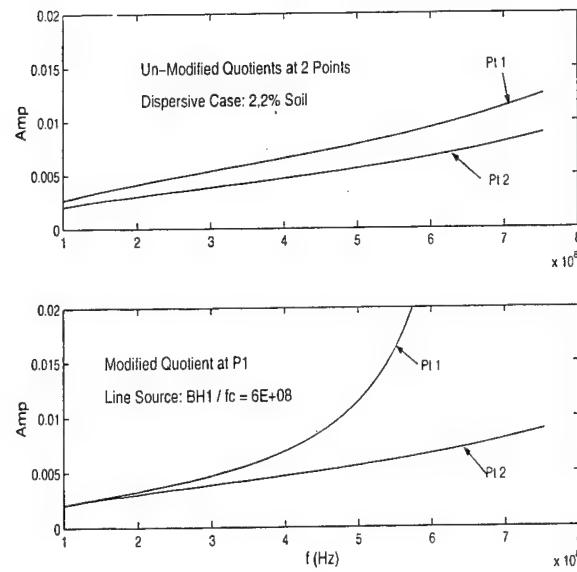
15

Figure 7: FDTD: Non-dispersive Case.



Figure 8: FDTD: Dispersive Case.

16

It is interesting to compare qualitatively the reflected waves measured at the **same** point for the two cases. The graphs of the two reflected waves are shown in Figure 9. They are not only qualitatively the same, but they can hardly be distinguished from each other. (We have purposely plotted fewer points on one of the graphs for clarity.) A portion of this plot has been enlarged and is shown in Figure 10. Here we can start to discern some systematic differences. This suggests that a criterion or a measure to distinguish a non-dispersive medium from a dispersive medium must rely on information in the reflected wave measured at more than one point. In other words, it is not the shape of the reflected wave measured at one point that will tell us something about the medium, but rather the relationship between shapes of the reflected waves measured at different points that will.
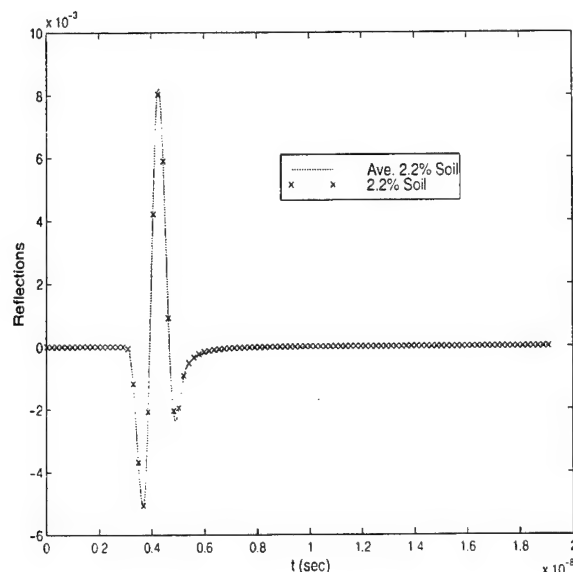


Figure 9: Reflected waves measured at the same point, one for a non-dispersive half space and one for a dispersive half space.

## IV-3. FDTD: Dispersiveness of a Checker-Board Dispersive Medium

In this experiment, we consider a half space made up of different dispersive material arranged in a checker-board-like pattern as shown in Figure 11.
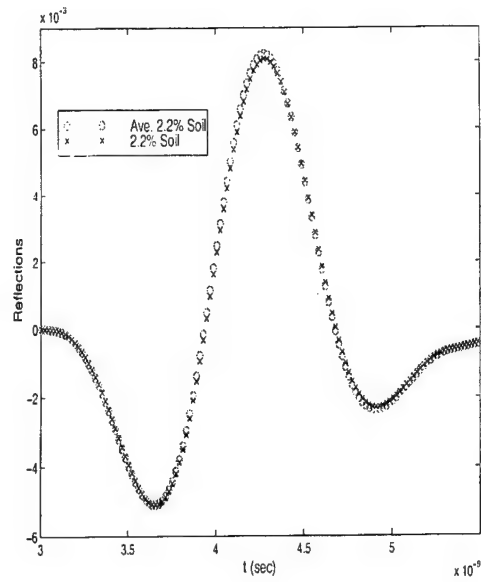
17

Figure 10: A zoom-in version of reflected waves measured at the same point, one for a non-dispersive half space and one for a dispersive half space.
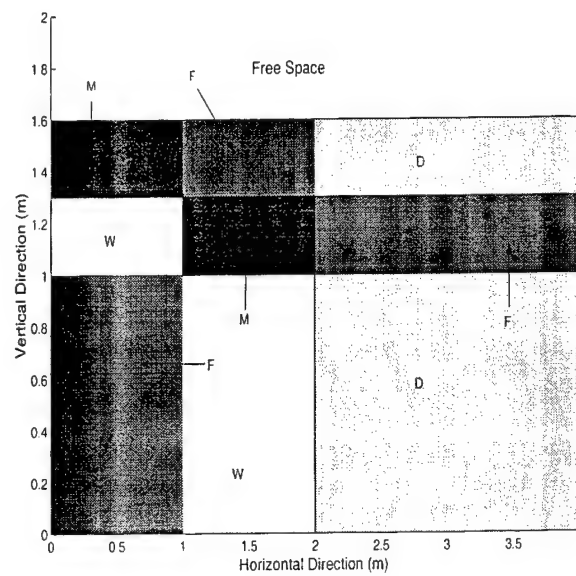


Figure 11: A Checker-Board-Like Half Space. Legends: F = Foliage, D = 0.0% Moisture Soil, M = 2.2% Moisture Soil, W = Wood.

Specifically, it consists of wood, foliage, and two types of soil, and are arranged in some random order. The media parameters as a functions of frequency are shown in Figure 3 and in Figure 4. The actual formulas for generating these graphs can be found in [7].

The incident field $E^{inc}$ and the total field $E_z$ at two related points $(x, y)$ and $(x_\lambda, y_\lambda)$ (as defined in Section III) are shown Figure 12 and Figure 13 respectively. Again, $\lambda$ is taken to be 2.
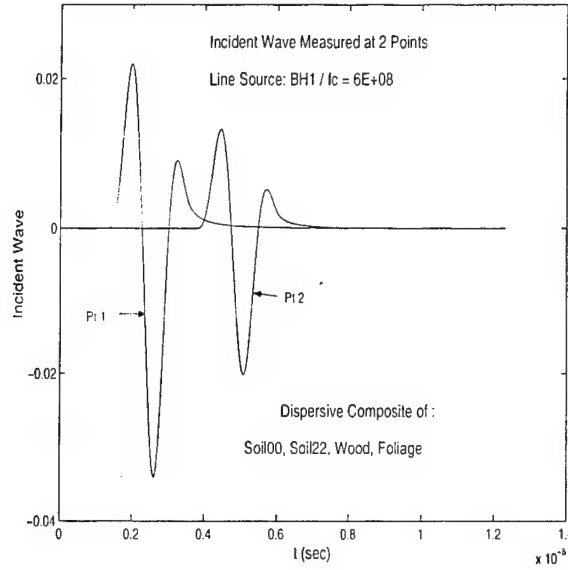


Figure 12: Composite Half Space: Incident field at $(x, y)$ and $(x_\lambda, y_\lambda)$. $\lambda = 2$.

The calculated reflected field $E^R$ at the same two points are shown in Figure 14

As in the previous numerical experiment, the quantities $\frac{E^R(\omega, x, y)}{I(\omega)}$ and $\frac{E^R(\omega, x_\lambda, y_\lambda)}{I(\omega)}$ are calculated and displayed in the top plot in Figure 15. Again the bottom plot displays the graphs of the scaled quotient $\frac{E^R(\lambda\omega, x, y)}{\lambda I(\lambda\omega)}$ and the (unscaled) quotient $\frac{E^R(\omega, x_\lambda, y_\lambda)}{I(\omega)}$.

The results are comparable to that for the previous experiment involving a simple dispersive half space. In particular, we cannot conclude from this calculation that *this* composite half space is non-dispersive.
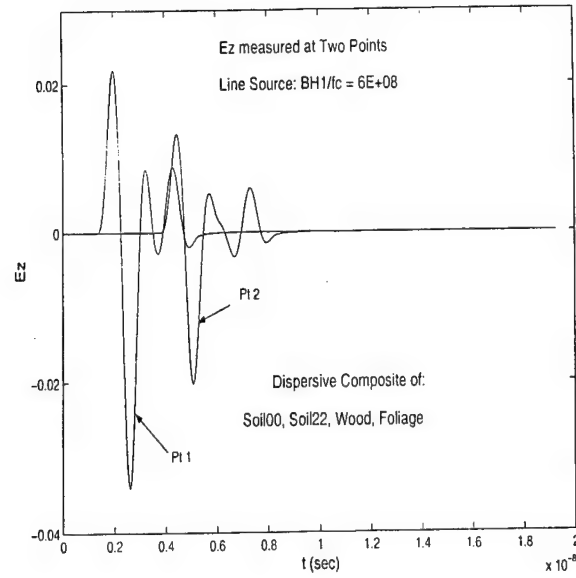
19

Figure 13: Composite Half Space: Total field at $(x, y)$ and $(x_\lambda, y_\lambda)$. $\lambda = 2$.
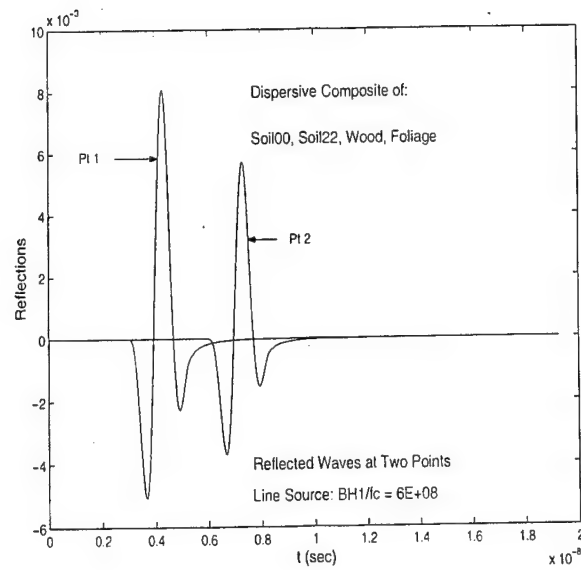


Figure 14: Composite Half Space: Reflected field at $(x, y)$ and $(x_\lambda, y_\lambda)$. $\lambda = 2$.
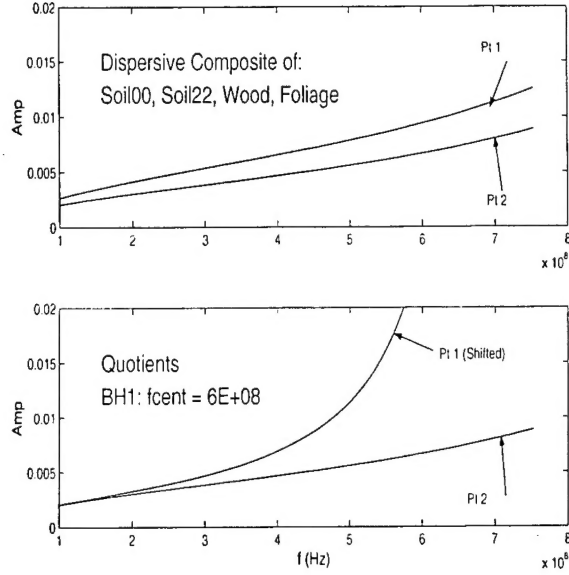
20

Figure 15: Quotients (see text) for Checker-board Medium.

# IV. Conclusion

An effectively non-dispersive medium can be defined subjectively as one in which reflected waves from the medium are characteristic of those from a non-dispersive one. We have proposed a simple measure, based on the use of a line source and the ensuing reflected waves from the medium (a half space), to decide if the medium is non-dispersive. The measure has value is zero when the medium is non-dispersive. We tested the measure on simulated data and showed that it can distinguish a known dispersive medium from a known non-dispersive medium, even though the reflected waves from the two are apparently indistinguishable. In applying the measure to a problem involving a fairly composite dispersive medium, we were able to conclude definitively that the medium is dispersive. Since the proposed measure can quantify a medium's departure from non-dispersiveness, it may be used to quantify its effective non-dispersiveness (a subjective notion) based on the magnitude of the departure. Thus, while the composite dispersive medium we considered is dispersive under our measure, it could well be considered effectively non-dispersive in some applications, because the departure from non-dispersiveness may not be deemed significant.

21

Once we have determined a half space is non-dispersive, it is possible to identify the medium parameters using least squares, cross-correlation, or transform methods. We also believe the approach here can be extended to a conductive half space. These are our next goals.

## Acknowledgment

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